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A Class of Convolution Integral Equations

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A new class of convolution integral equations whose kernels involve an H-function of several variables, which is defined by a multiple contour integral of the Mellin–Barnes type, is solved. It is also indicated how the main theorem can be specialized to derive a number of (known or new) results on convolution integral equations involving simpler special functions of interest in problems of applied mathematics and mathematical physics.

1. INTRODUCTION

In recent years several authors have made significant contributions to the theory of convolution integral equations whose kernels involve certain special functions of one and more variables (see, for example, Srivastava and Buschman [6], Srivastava [5], and Buschman, Koul and Gupta [1]). Indeed, much of the earlier work on the subject of convolution integral equations with special function kernels has been systematically presented in the book by Srivastava and Buschman [7]. The object of the present paper

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is to develop extensions of these results to hold for the convolution integral equation

$$\int_0^x (x-t)^{\rho-1} e^{\alpha(x-t)} \mathbf{H}\begin{pmatrix} \omega_1(x-t) \\ \vdots \\ \omega_r(x-t) \end{pmatrix} f(t)\,dt$$

$$= g(x), \qquad 0 \leqslant x < \infty, \qquad \operatorname{Re}(\rho) > 0, \tag{1.1}$$

where g is prescribed such that

$$g^{(l)}(0) = 0, \qquad l = 0, 1, 2, \ldots, n-1,$$

n being a positive integer, f is an unknown function to be determined, and the kernel involves a special class of the H-function of r variables, defined by Srivastava and Panda [8, p. 271, Eq. (4.1) *et seq.*] by

$$\mathbf{H}\begin{pmatrix} z_1 \\ \vdots \\ z_r \end{pmatrix} = \mathbf{H}^{0,0\,:\,(1,\nu');\ldots;(1,\nu^{(r)})}_{A,C\,:\,[B',D'+1];\ldots;[B^{(r)},D^{(r)}+1]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: \\ [(c):\psi',\ldots,\psi^{(r)}]: \end{matrix}\right.$$

$$\left.\begin{matrix} [(b'):\phi'];\cdots; & [(b^{(r)}):\phi^{(r)}]; \\ [0:1],[(d'):\delta'];\cdots;[0:1], & [(d^{(r)}):\delta^{(r)}]; \end{matrix}\; z_1,\ldots,z_r\right)$$

$$= \frac{1}{(2\pi i)^r}\int_{\mathscr{L}_1}\cdots\int_{\mathscr{L}_r} \Phi_1(\zeta_1)\cdots\Phi_r(\zeta_r)\,\Psi(\zeta_1,\ldots,\zeta_r)$$

$$\times\,\Gamma(-\zeta_1)\cdots\Gamma(-\zeta_r)\,z_1^{\zeta_1}\cdots z_r^{\zeta_r}\,d\zeta_1\cdots d\zeta_r, \quad i=\sqrt{-1}, \tag{1.2}$$

or, equivalently, by

$$\mathbf{H}\begin{pmatrix} z_1 \\ \vdots \\ z_r \end{pmatrix} = \sum_{m_1,\ldots,m_r=0}^{\infty} \Phi_1(m_1)\cdots\Phi_r(m_r)\,\Psi(m_1,\ldots,m_r)$$

$$\times\,\frac{(-z_1)^{m_1}}{m_1!}\cdots\frac{(-z_r)^{m_r}}{m_r!}, \tag{1.3}$$

where

$$\Phi_k(\zeta) = \frac{\prod_{j=1}^{\nu^{(k)}} \Gamma(1-b_j^{(k)}+\phi_j^{(k)}\zeta)}{\prod_{j=1}^{D^{(k)}} \Gamma(1-d_j^{(k)}+\delta_j^{(k)}\zeta)\prod_{j=\nu^{(k)}+1}^{B^{(k)}} \Gamma(b_j^{(k)}-\phi_j^{(k)}\zeta)},$$

$$k = 1,\ldots,r, \tag{1.4}$$

$$\Psi(\zeta_1,\ldots,\zeta_r) = \left\{\prod_{j=1}^{A}\Gamma\left(a_j-\sum_{k=1}^{r}\theta_j^{(k)}\zeta_k\right)\prod_{j=1}^{C}\Gamma\left(1-c_j+\sum_{k=1}^{r}\psi_j^{(k)}\zeta_k\right)\right\}^{-1}, \tag{1.5}$$

an empty product in (1.4) and (1.5) is to be interpreted as 1, the parameters a_j $(j=1,\ldots,A)$, $b_j^{(k)}$ $(j=1,\ldots,B^{(k)};\ k=1,\ldots,r)$, c_j $(j=1,\ldots,C)$, and $d_j^{(k)}$ $(j=1,\ldots,D^{(k)};\ k=1,\ldots,r)$ are suitable real or complex numbers, the associated coefficients $\theta_j^{(k)}$ $(j=1,\ldots,A)$, $\phi_j^{(k)}$ $(j=1,\ldots,B^{(k)})$, $\psi_j^{(k)}$ $(j=1,\ldots,C)$, $\delta_j^{(k)}$ $(j=1,\ldots,D^{(k)})$, $k=1,\ldots,r$, are positive (real) numbers, the integers A, $B^{(k)}$, C, $D^{(k)}$ and $\nu^{(k)}$ are constrained by $A\geqslant 0$, $0\leqslant \nu^{(k)}\leqslant B^{(k)}$, $C\geqslant 0$, $D^{(k)}\geqslant 0$, $k=1,\ldots,r$, and the contour $\mathscr{L}_k$ in the complex ζ_k-plane is of the Mellin–Barnes type which runs from $-i\infty$ to $i\infty$ with indentations, if necessary, in such a manner that all the poles of $\Gamma(-\zeta_k)$ are to the right, and those of $\Gamma(1-b_j^{(k)}+\phi_j^{(k)}\zeta_k)$, $j=1,\ldots,\nu^{(k)}$, to the left, of $\mathscr{L}_k$, the various parameters being so restricted that these poles are all simple and none of them coincide.. If we let

$$\Lambda_k = 1-\sum_{j=1}^{A}\theta_j^{(k)}+\sum_{j=1}^{\nu^{(k)}}\phi_j^{(k)}-\sum_{j=\nu^{(k)}+1}^{B^{(k)}}\phi_j^{(k)} - \sum_{j=1}^{C}\psi_j^{(k)}-\sum_{j=1}^{D^{(k)}}\delta_j^{(k)}, \qquad k=1,\ldots,r, \tag{1.6}$$

then it is fairly well known that, for $\Lambda_k>0$, $k=1,\ldots,r$, and with the points $z_k=0$, $k=1,\ldots,r$, being tacitly excluded, the multiple contour integral in (1.2) converges absolutely and defines an H-function of r variables, analytic in the sectors given by

$$|\arg(z_k)|<\tfrac{1}{2}\Lambda_k\pi, \qquad k=1,\ldots,r. \tag{1.7}$$

Note that, in view of (1.3), this multivariable H-function makes sense also when

$$\Omega_k\equiv\sum_{j=1}^{A}\theta_j^{(k)}+\sum_{j=1}^{B^{(k)}}\phi_j^{(k)}-\sum_{j=1}^{C}\psi_j^{(k)}-\sum_{j=1}^{D^{(k)}}\delta_j^{(k)}\leqslant 1, \qquad k=1,\ldots,r, \tag{1.8}$$

where each of the equalities holds true for suitably bounded values of $z_1,\ldots,z_r$.

For the sake of ready reference we list here the following definitions and results which will also be required in the course of our investigation.

I. The definition of the (unilateral) Laplace transform in the form (cf., e.g., [9]):

$$\mathscr{F}(s)=\int_0^\infty e^{-st}f(t)\,dt, \qquad \operatorname{Re}(s-\sigma)>0, \tag{1.9}$$

which may be written symbolically as

$$\mathscr{F}(s)=\mathscr{L}\{f(t): s\}, \tag{1.10}$$

or

$$f(t)=\mathscr{L}^{-1}\{\mathscr{F}(s):t\} \tag{1.11}$$

provided that $f(t)$ is continuous for $t \geqslant 0$ and of exponential order $e^{\sigma t}$ when $t \to \infty$.

Thus we readily have ([2, pp. 129, 131]; see also [9])

$$\mathscr{L}\{e^{-\lambda t}f(t):s\}=\mathscr{F}(s+\lambda), \qquad \operatorname{Re}(s+\lambda-\sigma)>0; \tag{1.12}$$

$$\mathscr{L}\{f^{(n)}(t):s\}=s^n\mathscr{F}(s), \qquad f^{(n)}(t)=\mathscr{L}^{-1}\{s^n\mathscr{F}(s):t\}, \tag{1.13}$$

provided that $f(t)\in\mathscr{C}^n$ for $0\leqslant t<\infty$, and

$$f(0)=f'(0)=\cdots=f^{(n-1)}(0)=0, \tag{1.14}$$

n being a positive integer.

II. The convolution theorem for Laplace's transform:

$$\mathscr{L}\{(f*g)(t):s\}=\mathscr{F}(s)\,\mathscr{G}(s)=\mathscr{L}\{f(t):s\}\;\mathscr{L}\{g(t):s\}, \tag{1.15}$$

where

$$(f*g)(t)=\int_0^t f(t-\tau)\,g(\tau)\,d\tau=(g*f)(t). \tag{1.16}$$

III. The following special case of a known Laplace transform pair [4, p.120, Eq. (3.14)]:

$$\mathscr{L}\left\{t^{\rho-1}\,\mathbf{H}\begin{pmatrix}\omega_1 t\\ \vdots\\ \omega_r t\end{pmatrix}:s\right\}=s^{-\rho}\mathscr{H}\begin{pmatrix}\omega_1/s\\ \vdots\\ \omega_r/s\end{pmatrix}, \quad \operatorname{Re}(\rho)>0, \quad \operatorname{Re}(s)>0, \tag{1.17}$$

which, in view of (1.12), may be rewritten at once as

$$\mathscr{L}\left\{t^{\rho-1}e^{\alpha t}\,\mathbf{H}\begin{pmatrix}\omega_1 t\\ \vdots\\ \omega_r t\end{pmatrix}:s\right\}=(s-\alpha)^{-\rho}\,\mathscr{H}\begin{pmatrix}\omega_1/(s-\alpha)\\ \vdots\\ \omega_r/(s-\alpha)\end{pmatrix},$$

$$\operatorname{Re}(\rho)>0, \quad \operatorname{Re}(s-\alpha)>0, \tag{1.18}$$

where, for convenience,

$$\mathscr{H}\begin{pmatrix} z_1 \\ \vdots \\ z_r \end{pmatrix} = \mathbf{H}^{0,1\,:\,(1,\nu');\ldots;(1,\nu^{(r)})}_{A+1,C\,:\,[B',D'+1];\ldots;[B^{(r)},D^{(r)}+1]} \left(\begin{matrix} [1-\rho:1,\ldots,1],[(a):\theta',\ldots,\theta^{(r)}]: \\ [(c):\psi',\ldots,\psi^{(r)}]: \end{matrix} \right.$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots; & [(b^{(r)}):\phi^{(r)}]; \\ [0:1],[(d'):\delta'];\ldots; & [0:1],[(d^{(r)}):\delta^{(r)}]; \end{matrix} z_1,\ldots,z_r \right). \tag{1.19}$$

2. Solution of the Integral Equation (1.1)

By the convolution theorem (1.15) and the Laplace transform pair (1.18), we find from (1.1) that

$$(s-\alpha)^{-\rho}\,\mathscr{H}\begin{pmatrix} \omega_1/(s-\alpha) \\ \vdots \\ \omega_r/(s-\alpha) \end{pmatrix} \mathscr{F}(s) = \mathscr{G}(s), \tag{2.1}$$

whence

$$\mathscr{F}(s) = (s-\alpha)^{\rho}\,\mathscr{G}(s) \left\{ \mathscr{H}\begin{pmatrix} \omega_1/(s-\alpha) \\ \vdots \\ \omega_r/(s-\alpha) \end{pmatrix} \right\}^{-1}, \tag{2.2}$$

provided that $\min\{\mathrm{Re}(\rho), \mathrm{Re}(s-\alpha)\} > 0$ and, of course, $\mathscr{H}(\cdots) \neq 0$.

Making use of an expansion formula analogous to (1.3), derivable also from definitions [8, p. 271, Eq. (4.1)], we can write

$$\mathscr{H}\begin{pmatrix} \omega_1/(s-\alpha) \\ \vdots \\ \omega_r/(s-\alpha) \end{pmatrix} = \sum_{m_1,\ldots,m_r=0}^{\infty} \Delta(m_1,\ldots,m_r)\,\Gamma(\rho+m_1+\cdots+m_r)$$

$$\times (s-\alpha)^{-m_1-\cdots-m_r} \frac{(-\omega_1)^{m_1}}{m_1!} \cdots \frac{(-\omega_r)^{m_r}}{m_r!}$$

$$= \sum_{M=0}^{\infty} \gamma_M (s-\alpha)^{-M}, \tag{2.3}$$

where

$$\gamma_M = (-1)^M\,\Gamma(\rho+M) \sum_{m_1+\cdots+m_r=M} \Delta(m_1,\ldots,m_r)\,\frac{\omega_1^{m_1}}{m_1!}\cdots\frac{\omega_r^{m_r}}{m_r!} \tag{2.4}$$

and

$$\Delta(m_1,\ldots, m_r) = \Phi_1(m_1) \cdots \Phi_r(m_r)\ \Psi(m_1,\ldots, m_r) \tag{2.5}$$

in terms of $\Phi_k(\zeta)$ and $\Psi(\zeta_1,\ldots, \zeta_r)$ defined by (1.4) and (1.5), respectively.

Now let μ denote the least M for which $\gamma_M \neq 0$ and assume that the parameters are so constrained that the coefficients γ_M are all defined for

$$M = \mu,\ \mu+1,\ \mu+2,\ldots.$$

Then the series

$$\left\{\sum_{M=0}^{\infty} \gamma_{M+\mu}(s-\alpha)^{-M}\right\}^{-1} = \sum_{N=0}^{\infty} \xi_N(s-\alpha)^{-N} \tag{2.6}$$

converges (absolutely) for $|s-\alpha| > \varepsilon$ because the series to be reciprocated has the leading coefficient $\gamma_\mu \neq 0$, and hence the function has no zeros in the interval

$$|s-\alpha|^{-1} < \varepsilon$$

for some $\varepsilon > 0$.

Under these restrictions, (2.2) becomes

$$\mathscr{F}(s) = \left\{(s-\alpha)^{-(n-\mu-\rho)} \sum_{N=0}^{\infty} \xi_N(s-\alpha)^{-N}\right\}\{(s-\alpha)^n\,\mathscr{G}(s)\}, \tag{2.7}$$

which can be interpreted conveniently and simply as a convolution integral provided

$$\mathrm{Re}(\rho) < n-\mu,$$

n being a positive integer. We are thus led to our solution of the convolution integral equation (1.1) contained in the following

THEOREM. *With* Λ_k *defined by* (1.6), *let* $\Lambda_k > 0$ *and* $|\arg(\omega_k)| < \frac{1}{2}\Lambda_k\pi$, $k = 1,\ldots, r$ [*or, alternatively, let each of the inequalities in* (1.8) *hold true*]. *Also let* $0 < \mathrm{Re}(\rho) < n-\mu$, *and*

$$g(0) = g'(0) = \cdots = g^{(n-1)}(0) = 0, \tag{2.8}$$

and suppose that $\mathscr{L}\{g^{(n)}(t){:}s\}$ *exists for* $n = 1, 2, 3,\ldots$.

Then the convolution integral equation (1.1) *has for its solution*

$$f(x) = \int_0^x (x-t)^{n-\mu-\rho-1}\, e^{\alpha(x-t)} R_{n-\mu-\rho}(x-t)$$

$$\times (D_t-\alpha)^n\,\{g(t)\}, \qquad D_t \equiv \frac{d}{dt}, \tag{2.9}$$

where

$$R_\eta(z) = \sum_{N=0}^{\infty} \frac{\xi_N z^N}{\Gamma(N+\eta)}, \tag{2.10}$$

and the coefficients ξ_N *are given by the recursion formulas*

$$\xi_0 \gamma_\mu = 1, \qquad \sum_{l=0}^{q} \xi_l \gamma_{q+\mu-l} = 0, \qquad q = 1, 2, 3, \ldots, \tag{2.11}$$

or, explicitly, by

$$\xi_N = (-1)^N (\gamma_\mu)^{-N-1} \det \begin{bmatrix} \gamma_{\mu+1} & \gamma_\mu & 0 & 0 \cdots 0 \\ \gamma_{\mu+2} & \gamma_{\mu+1} & \gamma_\mu & 0 \cdots 0 \\ \vdots & \vdots & & \\ \gamma_{\mu+N} & \gamma_{\mu+N-1} & & \cdots \gamma_{\mu+1} \end{bmatrix} \tag{2.12}$$

in terms of the coefficients γ_M *defined by* (2.4), μ *being the least* M *for which* $\gamma_M \neq 0$.

Remark. From the power series expansion in (2.10) it is easily seen that the R-function is an entire function and that the *resolvent kernel* has the expression

$$\begin{aligned} K_{n-\mu-\rho}(z) &= z^{n-\mu-\rho-1} e^{\alpha z} R_{n-\mu-\rho}(z) \\ &= e^{\alpha z} \sum_{N=0}^{\infty} \xi_N \frac{z^{N+n-\mu-\rho-1}}{\Gamma(N+n-\mu-\rho)}, \end{aligned} \tag{2.13}$$

in which the series converges (absolutely) when $|z| > 0$.

3. Applications

By assigning suitable values to the parameters occurring in (1.2), our theorem can be applied to derive solutions of certain classes of convolution integral equations whose kernels involve various special functions of one and more variables. We list below some of these important special cases of the integral equation (1.1).

I. If we set each θ, ϕ, ψ and δ in (1.2) equal to 1, the multivariable H-function occurring in (1.1) will reduce immediately to the corresponding G-function of r variables (see, for example, [8, p. 273, Eq. (4.9)]), and we have the integral equation

$$\int_0^x (x-t)^{\rho-1} e^{\alpha(x-t)} \mathbf{G} \begin{pmatrix} \omega_1(x-t) \\ \vdots \\ \omega_r(x-t) \end{pmatrix} f(t)\,dt$$
$$= g(x), \qquad 0 \leqslant x < \infty, \quad \operatorname{Re}(\rho) > 0, \tag{3.1}$$

whose solution is obtainable by similarly specializing (2.9).

II. For $A = C = 0$, the multivariable H-function occurring in (1.1) degenerates into the product of r *distinct* H-functions of Fox [3, p. 408], and we are led to the convolution integral equation

$$\int_0^x (x-t)^{\rho-1} e^{\alpha(x-t)} f(t)$$
$$\times \prod_{k=1}^{r} \left\{ \mathbf{H}^{1,\nu^{(k)}}_{B^{(k)},D^{(k)}+1} \left[\omega_k(x-t) \middle| \begin{matrix} [(b^{(k)}):\phi^{(k)}] \\ [0:1],\ [(d^{(k)}):\delta^{(k)}] \end{matrix} \right] \right\} dt$$
$$= g(x), \qquad 0 \leqslant x < \infty, \qquad \operatorname{Re}(\rho) > 0, \tag{3.2}$$

whose solution would follow readily from our theorem by setting $A = C = 0$.

III. If we further let each ϕ and δ occurring in (3.2) equal 1, the H-functions would reduce to Meijer's G-functions (and also to MacRobert's E-functions) (see [2, pp. 373–374]), and we shall obtain a convolution integral equation involving the product of r *distinct* G or E functions.

IV. If in (1.1) we put $A = 0$ and $\nu^{(k)} = B^{(k)}$, $k = 1, \ldots, r$, and apply a known relationship [8, p. 272, Eq. (4.7)], we obtain the integral equation

$$\int_0^x (x-t)^{\rho-1} e^{\alpha(x-t)} \mathbf{F}^{0:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}} \begin{pmatrix} -\omega_1(x-t) \\ \vdots \\ -\omega_r(x-t) \end{pmatrix} f(t)\,dt$$
$$= \frac{\prod_{j=1}^{C} \Gamma(1-c_j) \prod_{j=1}^{D'} \Gamma(1-d_j') \cdots \prod_{j=1}^{D^{(r)}} \Gamma(1-d_j^{(r)})}{\prod_{j=1}^{B'} \Gamma(1-b_j') \cdots \prod_{j=1}^{B^{(r)}} \Gamma(1-b_j^{(r)})}$$
$$\times g(x), \qquad 0 \leqslant x < \infty, \quad \operatorname{Re}(\rho) > 0, \tag{3.3}$$

where

$$\mathbf{F}^{0:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}} \begin{pmatrix} z_1 \\ \vdots \\ z_r \end{pmatrix}$$
$$= \mathbf{F}^{0:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}} \left(\begin{matrix} \overline{\qquad\qquad\qquad}: \\ [1-(c):\psi',\ldots,\psi^{(r)}]: \end{matrix} \right.$$
$$\left. \begin{matrix} [1-(b'):\phi'];\ldots;[1-(b^{(r)}):\phi^{(r)}]; \\ [1-(d'):\delta'];\ldots;[1-(d^{(r)}):\delta^{(r)}]; \end{matrix}\ z_1,\ldots,z_r \right) \tag{3.4}$$

is a special case of the (Srivastava–Daoust) generalized Lauricella hypergeometric function of r variables.

V. By setting $C=0$, the integral equation (3.3) can be further reduced to a convolution integral equation involving the product of r *distinct* (Wright's) generalized hypergeometric functions.

VI. If in (3.3) we equate each ϕ, ψ and δ to 1, we shall obtain a convolution integral equation involving a class of *confluent* hypergeometric functions of r variables. In particular, if we *further* set

$$B^{(k)}=1, \quad C=1, \quad D^{(k)}=0, \quad b_1^{(k)}=1-\beta_k, \quad c_1=1-\rho, \qquad k=1,\dots,r,$$

we arrive at the integral equation

$$\int_0^x (x-t)^{\rho-1}\, e^{\alpha(x-t)}\Phi_2^r[\beta_1,\dots,\beta_r;\rho;\, -\omega_1(x-t),\dots,\, -\omega_r(x-t)] \times f(t)\,dt=\frac{\Gamma(\rho)}{\Gamma(\beta_1)\cdots\Gamma(\beta_r)}g(x), \qquad 0\leqslant x<\infty, \quad \operatorname{Re}(\rho)>0, \tag{3.5}$$

where Φ_2^r denotes a *confluent* hypergeometric function of r variables, defined by (cf. [2, p. 385])

$$\Phi_2^r[\alpha_1,\dots,\alpha_r;\beta;z_1,\dots,z_r]=\frac{\Gamma(\beta)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_r)} \times \sum_{m_1,\dots,m_r=0}^{\infty}\frac{\Gamma(\alpha_1+m_1)\cdots\Gamma(\alpha_r+m_r)}{\Gamma(\beta+m_1+\cdots+m_r)}\frac{z_1^{m_1}}{m_1!}\cdots\frac{z_r^{m_r}}{m_r!}, \quad \max\{|z_1|,\dots,|z_r|\}<\infty. \tag{3.6}$$

The solution of the integral equation (3.5), readily obtainable by suitably applying our theorem, corresponds precisely to the solution given earlier by Srivastava [5, p. 254, Eq. (15)], who indeed also discussed several interesting special or confluent cases of (3.5).

We conclude by remarking that each of the E, F, G and H functions occurring in the kernels of the integral equations considered in this paper can be reduced to a wide variety of simpler special functions of mathematical physics and applied mathematics (and to various products and other combinations of such functions). Our results can thus find many more applications than those listed above.

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